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Moshe Tur^a

^a The Department of Electronics, The Weizmann Institute of Science, Rehovot, Israel

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Reflection at the Boundary Between Glass and Cholesteric Liquid Crystals†

MOSHE TUR

The Department of Electronics, The Weizmann Institute of Science, Rehovot, Israel

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Reflection at the interface between glass and cholesteric liquid crystals is theoretically investigated for the case of normal incidence using Oseen-DeVries' model. Attention is focused on the case where the average index of refraction of the liquid crystal differs from the glass refractive index. It is then found that the general shape of the reflection spectrum is highly dependent on the polarization of the incident beam. Thus, for example, two incident waves which are linearly polarized in different directions will generally have their peak reflections at different wavelengths. The shape of the spectrum also depends, for a given liquid crystal, on the refractive index of the glass.

Light propagation through cholesteric liquid crystals in their Grandjean plane texture was investigated by several authors.¹⁻¹⁰ Complete analytical solutions in closed form were derived only for the case of normal incidence.^{1-3,5} The general propagation problem was also treated both numerically⁷⁻⁹ and analytically.¹⁰ In the following, only the normal incidence case will be considered.

DeVries² gives general expressions for the normal modes (i.e. modes of propagation) in the direction of the spiral axis. Calculations of the reflection coefficient from interfaces such as glass-liquid crystal or liquid crystal-glass as well as from a finite liquid crystal slab between two glass media are also carried out in DeVries' paper, but only for the case where the index of refraction of the glass equals the average index of refraction of the liquid crystal material. On the other hand, Conners³ solves the boundary value problem for a glass of arbitrary index of refraction but only for one specific wave-

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length, namely: $\lambda = p\sqrt{\epsilon}$ (p is the pitch and ϵ is the average dielectric constant of the liquid crystal, see Eq. (2c)).

In this paper we treat this same boundary value problem without any constraint either on the glass' index of refraction or the wavelength of the incident beam. After reviewing DeVries' work in some detail, we solve numerically for the reflection coefficient at the interface glass-liquid crystal. The resulting graphs indicate that the shapes of the spectrum of the reflected light is highly dependent on the incident polarization. For a given liquid crystal and a given polarization, it depends on the index of refraction of the glass. We then proceed by deriving approximate solutions of the boundary conditions equations. The shapes of the various reflection spectra are discussed in light of these approximations. Finally, we consider the reflection from a finite slab of liquid crystal.

1 OSEEN-DEVRIES THEORY

According to Oseen and DeVries, the cholesteric liquid crystal has a uniformly twisted nematic structure, the helical axis being perpendicular to the nematic symmetry axis. Thus the dielectric properties are described by a tensor that changes its orientation with the twist.

For the treatment of light propagation in this system, it is convenient to use two coordinate systems (Figure 1): the laboratory system x, y, z with the z -axis along the helical axis and a local coordinate system ξ, η, ζ with the ζ -axis parallel to the z -axis. The other two axes are positioned in such a way that the dielectric tensor is diagonal in it with principal values $\epsilon_1, \epsilon_2, \epsilon_3$ along the axes ξ, η, ζ , the angle β between the ξ -axis and the x -axis varies linearly with z : $\beta = 2\pi z/p$, p is the pitch of the structure. It is positive for a right-handed cholesteric and negative for a left-handed one. It will be assumed the $p > 0$ and $\epsilon_2 > \epsilon_1$. Since we deal exclusively with propagation along the spiral axis, the value of ϵ_3 need not be specified. According to Refs. 8 and 9: $\epsilon_1 = \epsilon_3$.

DeVries' solutions for the modes of propagation in the $\{\xi, \eta, \zeta\}$ frame are elliptically polarized waves⁶ which have the following form:

$$\begin{matrix} E_\xi = A \\ E_\eta = iB \end{matrix} \left\{ \exp \left[2\pi i \left(\frac{t}{T} - \frac{mz}{\lambda} \right) \right] \right. \quad (1)$$

λ is the wavelength in free space and $1/T$ is the optical frequency. Expressions for m and B/A are usually given in terms of the reduced quantities $m' = m/\sqrt{\epsilon}$, $\lambda' = \lambda/p\sqrt{\epsilon}$:

$$m' = \pm \{1 + (\lambda')^2 \pm [4(\lambda')^2 + \alpha^2]^{1/2}\}^{1/2} \quad (2a)$$

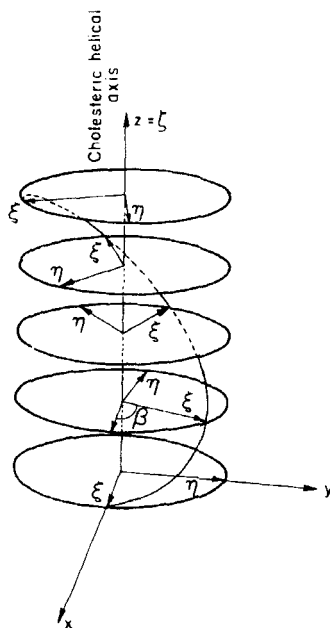


FIGURE 1 Definition of the laboratory and local coordinate systems used to discuss light propagation along the helix axis of the cholesteric liquid crystal (after Ref. 10). The z -axis is the cholesteric helical axis.

$$f = \frac{B}{A} = \frac{1 - \alpha - (m')^2 - (\lambda')^2}{2m'\lambda'} \quad (2b)$$

$$\varepsilon = \frac{\varepsilon_1 + \varepsilon_2}{2}; \alpha = \frac{\varepsilon_2 - \varepsilon_1}{2\varepsilon} \quad (2c)$$

ε is the average dielectric constant for normal incidence. α measures the birefringence of the liquid crystal. α is assumed here to be positive. Usually $\alpha < 0.1$.¹¹ The liquid crystal can therefore support four modes of propagation parallel to the axis of the structure: two in each direction.

Expressions for the magnetic field were also given by DeVries:

$$H_\xi = -\frac{\sqrt{\varepsilon}}{q} E_\eta \quad (3a)$$

$$H_\eta = q\sqrt{\varepsilon} E_\xi \quad (3b)$$

$$q = \lambda'f + m' \quad (3c)$$

The two modes which propagate in the $+z$ direction are characterized by two different values for m' :

$$m'_1 = \begin{cases} \{1 + (\lambda')^2 - [4(\lambda')^2 + \alpha^2]^{1/2}\}^{1/2}; & \lambda' \leq (1 - \alpha)^{1/2} \\ -i\{[4(\lambda')^2 + \alpha^2]^{1/2} - 1 - (\lambda')^2\}^{1/2}; & (1 - \alpha)^{1/2} < \lambda' < (1 + \alpha)^{1/2} \\ -\{1 + (\lambda')^2 - [4(\lambda')^2 + \alpha^2]^{1/2}\}^{1/2}; & (1 + \alpha)^{1/2} \leq \lambda' \end{cases} \quad (4a)$$

$$m'_2 = \{1 + (\lambda')^2 + [4(\lambda')^2 + \alpha^2]^{1/2}\}^{1/2} \quad (4b)$$

m' , f and q for these modes appear in Figure 2 as functions of λ' (α was taken to be 0.05). The abscissa is confined to those wavelengths which will be of interest later on. Note in Figure 2 that for $(1 - \alpha)^{1/2} < \lambda' < (1 + \alpha)^{1/2}$, m'_1 , f_1 and q_1 are purely imaginary. As f_2 and q_2 are quite close to -1 and 1 respectively, only their deviations from these values (multiplied by 200) are displayed in Figure 2.

The mode described by m'_2 will be called the second mode. For $\lambda' \gg \alpha$ it appears in the laboratory frame almost like a left-handed circularly

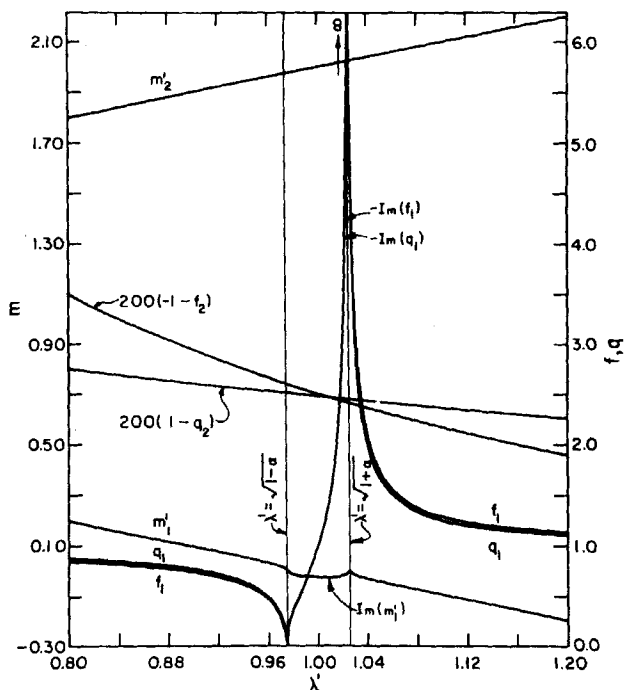


FIGURE 2 The wavelength dependence of the various parameters which define the two modes of propagation in the $+z$ direction. $f_{1,2}$ and $m_{1,2}$ are respectively the ellipticities and "refractive indices"² of the modes in the $\xi\eta\zeta$ frame. $q_{1,2}$ determine the relation between the electric and magnetic vectors of the above modes. ($\text{Im}(\)$) denotes the imaginary part.)

polarized wave¹² whose index of refraction changes very slightly with λ' . On the other hand, the polarization and index of refraction of the first mode, characterized by m'_1 , are highly dependent on wavelength. Moreover, in the region:

$$1 - \alpha < (\lambda')^2 < 1 + \alpha \quad (5)$$

this mode turns out to be a standing wave, i.e. energy entering the material is reflected back. Concerning the wave itself, its amplitude is exponentially decreasing in the $+z$ direction. This effect is a first order Bragg reflection. The region of wavelengths (5) is called therefore the reflection band. Outside this region (but for $\lambda' \gg \alpha$) this mode approximates in the laboratory frame, a right-handed circularly polarized wave.

In evaluating the reflection either from the interface glass-liquid crystal or from the interface liquid crystal-glass, DeVries limited himself to the situation where $n = \sqrt{\epsilon}$, n is the index of refraction of the glass. In doing so he intended to avoid the normal dielectric reflection always encountered when light impinges on a boundary between two dielectric materials with two unequal refractive indices.

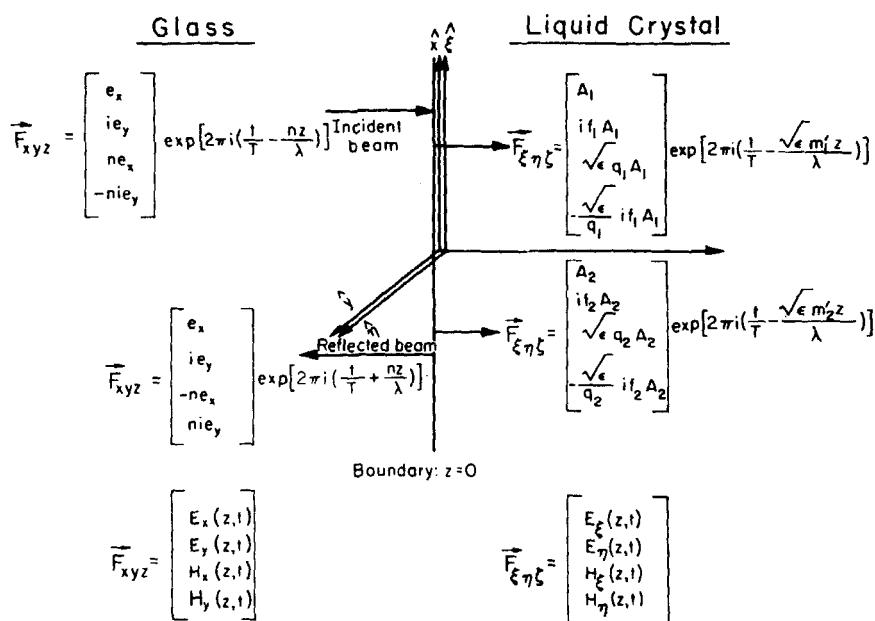


FIGURE 3 Illustration of the variables used in the calculation of the reflection from the boundary between a semi-infinite glass and a semi-infinite liquid crystal. The incident wave is partially reflected and partially transmitted. The transmitted wave is a superposition of the two modes (4a) and (4b).

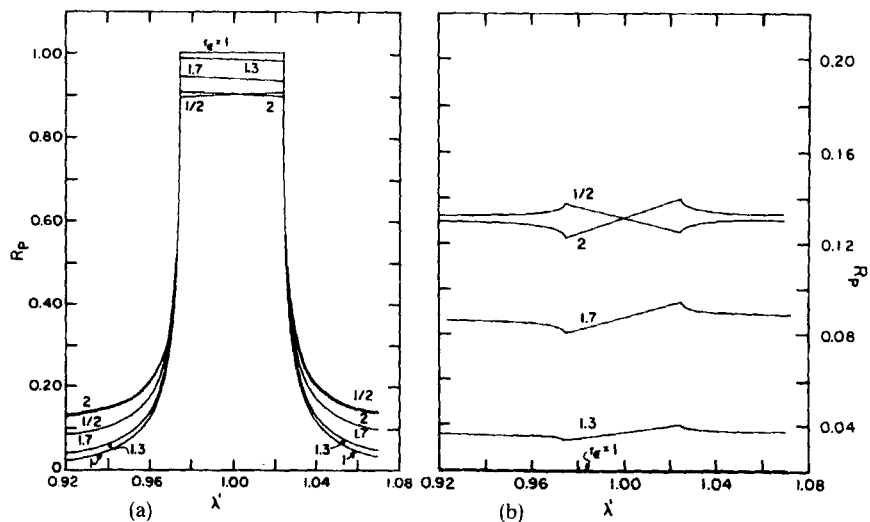


FIGURE 4 The reflection spectra for various values of r_e when $\alpha = 0.05$.

(a) The incident beam is right-handed circularly polarized.

(b) The incident beam is left-handed circularly polarized.

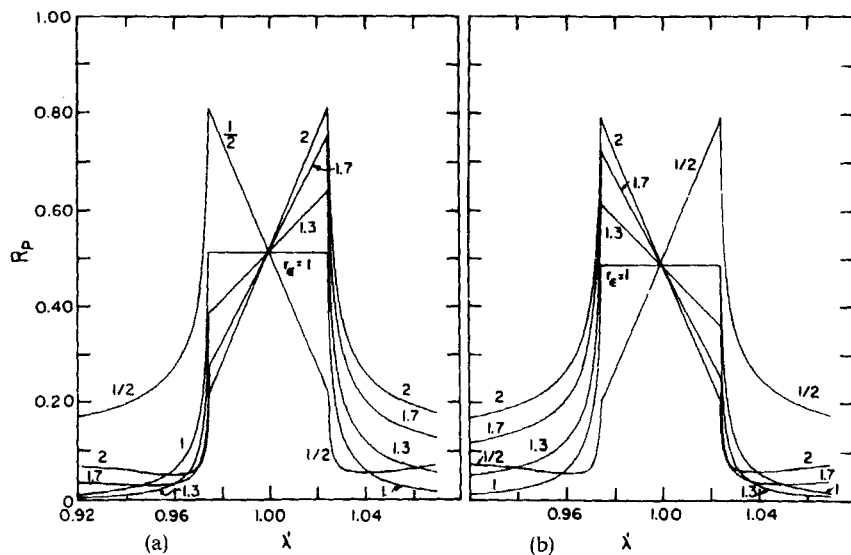


FIGURE 5 The reflection spectra for a linearly polarized incident wave.

$\alpha = 0.05$.

(a) linear polarization in the x-direction.

(b) linear polarization in the y-direction.

The graphs, designated by $r_e = 1$ in Figures 4 and 5, are the results of DeVries' calculations. These show that the reflection coefficient for a right-handed circularly polarized incident beam is approximately unity (for $\alpha = 0.05$ its exact value to four significant digits is 0.9998) throughout the reflection band. The reflected wave is right-handed circularly polarized. Outside the reflection band, the reflection coefficient decreases monotonically.

When the incident beam is left-handed circularly polarized the reflection coefficient is very small (less than 0.0003 for $\alpha = 0.05$) for all wavelengths $\lambda' \gg \alpha$, and the reflected wave can be neglected. It follows, that for every incident polarization the reflection coefficient is approximately constant throughout the reflection band. Its value depends on the amount of the right-handed circularly polarized component which is present in the incident beam.

II METHOD OF SOLUTION FOR $n \neq \sqrt{\epsilon}$

We shall solve now the glass-liquid crystal boundary problem for arbitrary n and $\sqrt{\epsilon}$. The configuration and notation appear in Figure 3. At the boundary glass-liquid crystal, i.e. at $z = 0$, the laboratory frame coincides with the rotating frame. Using the well known electromagnetic boundary conditions¹³ and referring to the notation of Figure 3 we get:

$$\begin{aligned} A_1 + A_2 &= e_x + e'_x \\ f_1 A_1 + f_2 A_2 &= e_y + e'_y \\ r_e \frac{f_1}{q_1} A_1 + r_e \frac{f_2}{q_2} A_2 &= e_y - e'_y \\ r_e q_1 A_1 + r_e q_2 A_2 &= e_x - e'_x \end{aligned} \tag{6}$$

r_e measures the refractive index mismatch between the glass and the liquid crystal:

$$r_e = \frac{\sqrt{\epsilon}}{n} \tag{7}$$

e_x and e_y are determined by the polarization and intensity of the incident beam while e'_x and e'_y characterize the reflected beam. The transmitted beam in the liquid crystal is represented by A_1 and A_2 . For a given incident polarization the quantities A_1 , A_2 , e'_x and e'_y as determined by the solution of Eq. (6) (for a given r_e) are functions of the wavelength of the incident wave.

A slight manipulation of set (6) results in a more convenient form:

$$\begin{aligned}\frac{f_1}{q_1}(q_1 + r_e)A_1 + \frac{f_2}{q_2}(q_2 + r_e)A_2 &= 2e_y \\ (r_e q_1 + 1)A_1 + (r_e q_2 + 1)A_2 &= 2e_x \\ e'_x &= A_1 + A_2 - e_x \\ e'_y &= (f_1 A_1 + f_2 A_2 - e_y)\end{aligned}\quad (8)$$

The set of Eqs. (8) was solved numerically (taking α to be 0.05) for various incident polarizations and for various values of r_e . The solutions for e'_x and e'_y were used to compute the reflection coefficient R_p :

$$R_p = \frac{(|e'_x|^2 + |e'_y|^2)}{(|e_x|^2 + |e_y|^2)} \quad (9)$$

RESULTS

Figures 4 and 5 describe R_p as a function of the wavelength λ' for incident beams with either circular or linear polarizations, as obtained from a numerical solution of Eq. (8).

When the incident beam is right-handed circularly polarized (see Figure 4a), R_p is rather constant throughout the reflection band and its value is less than unity for $r_e \neq 1$. On both sides of the reflection band R_p decreases monotonically, approaching a constant value which depends on r_e . This constant value is shown later to be approximately given by:

$$R_{p0} = \left[\frac{r_e - 1}{r_e + 1} \right]^2 \quad (10)$$

The reflection spectrum for a left-handed circularly polarized incident wave appears in Figure 4b. Within the reflection band and its vicinity the spectrum has a definite structure. Away from the reflection band, R_p is roughly constant, again approximately given by Eq. (10). For all wavelengths the reflected beam is mainly right-handed circularly polarized.

The laboratory frame x, y, z was chosen in such a way that its x and y directions coincide at $z = 0$ with the ξ and η directions of the rotating frame. Thus, the x - and y -axes are parallel to two principal axes of the liquid crystal dielectric tensor at $z = 0$. The reflection spectra for incident waves linearly polarized in these two directions appear in Figures 5a and 5b. For $r_e = 1$, R_p is constant throughout the reflection band. But when $r_e \neq 1$ the values of R_p at the two band edges are quite different. The behavior of R_p for wavelengths outside the reflection band will be discussed later.

III APPROXIMATE ANALYTIC SOLUTIONS

In this section we discuss approximate solutions of the set of Eqs. (8).

m_2 , f_2 and q_2 as defined by Eqs. (4b), (2b) and (3c) respectively, characterize the second mode in the liquid crystal. For $\lambda' \gg \alpha$ we can derive approximate formulas for these quantities:²

$$m'_2 = 1 + \lambda' + \frac{\alpha^2}{8\lambda'(1 + \lambda')} + 0\left(\frac{\alpha}{\lambda'}\right)^4 \quad (11)$$

$$f_2 = -1 - \frac{\alpha}{2\lambda'(1 + \lambda')} + 0\left(\frac{\alpha}{\lambda'}\right)^2 \quad (12)$$

$$q_2 = +1 - \frac{\alpha}{2(1 + \lambda')} + 0\left(\frac{\alpha}{\lambda'}\right)^2 \quad (13)$$

The deviations of f_2 and q_2 from -1 and 1 respectively, appear in Figure 2 for $\alpha = 0.05$.

In the following derivations we shall assume:

$$f_2 = -1; q_2 = 1 \quad (14)$$

A left-handed circularly polarized incident wave of unit intensity¹⁴ is given by:

$$\begin{bmatrix} Ex(z, t) \\ Ey(z, t) \end{bmatrix} = \frac{1}{\sqrt{2}} \cdot \begin{bmatrix} 1 \\ -i \end{bmatrix} \cdot \exp\left[2\pi i\left(\frac{t}{T} - \frac{nz}{\lambda}\right)\right] \quad (15)$$

so that:

$$e_x = \frac{1}{\sqrt{2}}; e_y = -\frac{1}{\sqrt{2}} \quad (16)$$

Substituting Eqs. (14) and (16) in Eq. (8), we obtain the following solution for the reflected wave:

$$\begin{bmatrix} Ex \\ Ey \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 - r_e \\ 1 + r_e \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -i \end{bmatrix} \exp\left[2\pi i\left(\frac{t}{T} + \frac{nz}{\lambda}\right)\right] \quad (17)$$

This reflected wave is right-handed circularly polarized. The reflection coefficient is given by:

$$Rp = \left[\frac{r_e - 1}{r_e + 1} \right]^2 \quad (18)$$

Although relation (18) does not describe the wavelength dependence of the reflection coefficient within the reflection band and its neighborhood, it may be used there as a first approximation since its deviation from the exact

reflection spectrum (Figure 4b) is quite small (within 10%). Outside the reflection band and for wavelengths not too close to it, Eq. (18) provides a much better approximation. Thus, for example, for $\lambda' = 0.92$ and $r_e = 2$, Rp as given in Figure 4b is 0.109 while Eq. (18) predicts 0.11.

A right-handed circularly polarized incident wave of unit intensity is represented by

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix} \exp \left[2\pi i \left(\frac{t}{T} - \frac{nz}{\lambda} \right) \right] \quad (19)$$

$$e_x = \frac{1}{\sqrt{2}}; e_y = \frac{1}{\sqrt{2}} \quad (20)$$

In the reflection band and its vicinity (cf. Figure 2):

$$q_1 = \lambda' f_1 + m'_1 \approx f_1 \quad (21)$$

When we solve Eq. (8) using Eqs. (20), (14), and (21) we get the following reflected wave:

$$\begin{aligned} \begin{bmatrix} E_x \\ E_y \end{bmatrix} &= \frac{4r_e(1 - f_1)}{(1 + r_e)^2(1 + f_1)} \cdot \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix} \exp \left[2\pi i \left(\frac{t}{T} + \frac{nz}{\lambda} \right) \right] \\ &+ \frac{1}{\sqrt{2}} \cdot \frac{1 - r_e}{1 + r_e} \cdot \begin{bmatrix} 1 \\ i \end{bmatrix} \exp \left[2\pi i \left(\frac{t}{T} + \frac{nz}{\lambda} \right) \right] \end{aligned} \quad (22)$$

The two terms in Eq. (22) represent, respectively, the right-handed and left-handed circularly polarized components of the reflected beam.

Inside the reflection band, f_1 is purely imaginary, and we can introduce the angle ϕ defined by:

$$e^{i\phi} = \frac{1 - f_1}{1 + f_1} \quad (23)$$

ϕ appears in Figure 6 as a function of λ' .

The reflection coefficient inside the reflection band is given by:

$$Rp = \left[\frac{4r_e}{(1 + r_e)^2} \right]^2 \left| \frac{1 - f_1}{1 + f_1} \right|^2 + \left[\frac{1 - r_e}{1 + r_e} \right]^2 = \frac{1 + 14r_e^2 + r_e^4}{(1 + r_e)^4} \quad (24)$$

According to Eq. (24), Rp should be constant throughout the reflection band while in Figure 4a Rp varies slightly within the band. The origin of this discrepancy is in our approximations (14) and (21). Nevertheless, the accuracy of Eq. (24) is quite good; e.g., for $r_e = 2$, Rp at the center of the band is 0.90 in both (24) and Figure 4a.

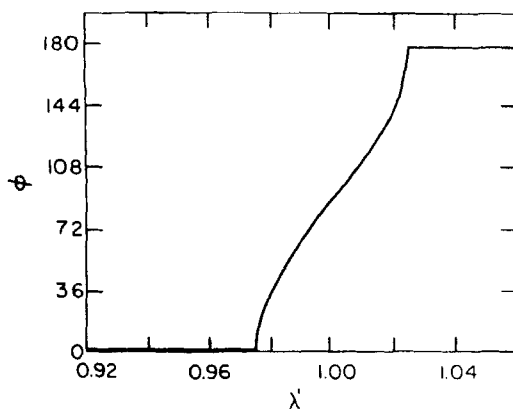


FIGURE 6 The wavelength dependence of the angle ϕ defined in Eq. (23). $\alpha = 0.05$.

Outside the reflection band, f_1 and q_1 tend to approach unity, see Figure 2. In this case R_p as calculated from Eq. (22) has a much simpler form:

$$R_p = \left[\frac{r_e - 1}{r_e + 1} \right]^2 = R_{p0} \quad (\text{cf. Eq. 10}) \quad (25)$$

and the reflected wave is left-handed circularly polarized.

IV DISCUSSION

The wave reflected from the interface glass-liquid crystal has two different sources: dielectric reflection at the boundary and internal (Bragg) reflection within the liquid crystal.

Dielectric reflection is due to the abrupt change in dielectric properties which the incident beam meets on passing the boundary between the glass and the liquid crystal. Since glass is optically isotropic while cholesteric liquid crystals are birefringent, dielectric reflection is always present even when $n = \sqrt{\epsilon}$. However, the birefringence is small ($\alpha < 0.1$) and quite often its contribution to the dielectric reflection can be neglected. In such cases light would be dielectrically reflected at the boundary glass-liquid crystal as if it were a boundary between two isotropic materials with a relative refractive index given by r_e .¹⁷

The reflected wave (17) and the second term in (22) represent this dielectric reflection. They vanish for $r_e = 1$ and their magnitude does not depend on λ' as long as dispersion is neglected. In the process of dielectric reflection, circularly polarized incident beams change their sense of rotation on reflection. The wave (17) and the second term in Eq. (22) are polarized in accordance with this rule.

That part of the incident beam which was not dielectrically reflected at the boundary, enters the liquid crystal. This transmitted wave is a superposition of the two modes (4a) and (4b). For wavelengths within the reflection band and its vicinity the energy which is carried in the form of the first mode (4a) is reflected back by the internal liquid crystal reflection mechanism. This reflected wave is right-handed circularly polarized. As a function of r_e , the magnitude of this internally reflected wave, attains its maximum for $r_e = 1$ because then almost all of the incident energy enters the liquid crystal.

When the incident beam is right-handed circularly polarized, the reflected wave whose source is the internal reflection is represented by the first term in Eq. (22). The amplitude of this reflected wave has a constant absolute value¹⁵ within the reflection band while its phase varies as in Figure 6.

Consider now a left-handed circularly polarized incident beam. The amplitudes of the two modes which constitute the transmitted wave in the liquid crystal can be found from (8) using the approximations (14):

$$A_1 = 0; A_2 = \frac{2}{1 + r_e} \quad (26)$$

Thus to this degree of approximation all the energy of the transmitted wave propagates in the form of the second mode and no internally reflected component is present in the reflected wave (17).

A more accurate solution of (8), which takes into account the deviations of f_2 and q_2 from -1 and 1 respectively, shows, however, that A_1 is small but nonzero. Within the reflection band and its vicinity the reflected wave includes therefore, a small internally reflected component in addition to the dielectrically reflected wave (17). These two right-handed circularly polarized components may interfere constructively or destructively according to their relative phase angle, which depends on λ' and on the sign of $1 - r_e$. The structure of the reflection spectrum in Figure 4b is a manifestation of this interference.

Once the reflection problem has been solved for circularly polarized incident beams, any other incident wave either polarized or unpolarized can be handled as a superposition of circular polarizations. This principle is demonstrated below by a few examples.

An incident beam linearly polarized in the x direction may be written as:

$$\begin{aligned} \begin{bmatrix} E_x \\ E_y \end{bmatrix} &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \exp \left[2\pi i \left(\frac{t}{T} - \frac{nz}{\lambda} \right) \right] \\ &= \frac{1}{\sqrt{2}} \left\{ \frac{1}{\sqrt{2}} \cdot \begin{bmatrix} 1 \\ i \end{bmatrix} + \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix} \right\} \exp \left[2\pi i \left(\frac{t}{T} - \frac{nz}{\lambda} \right) \right] \end{aligned} \quad (27)$$

Inside the reflection band, the reflected wave can be calculated from (17) and (22) using (27):

$$\begin{bmatrix} Ex \\ Ey \end{bmatrix} = \left\{ \left[\frac{2r_e e^{i\phi}}{(1+r_e)^2} + \frac{1}{2} \frac{1-r_e}{1+r_e} \right] \cdot \begin{bmatrix} 1 \\ -i \end{bmatrix} + \frac{1}{2} \cdot \frac{1-r_e}{1+r_e} \cdot \begin{bmatrix} 1 \\ i \end{bmatrix} \right\} \exp \left[2\pi i \left(\frac{t}{T} + \frac{nz}{\lambda} \right) \right] \quad (28)$$

R_p is given by:

$$R_p = 2 \left| \frac{2r_e e^{i\phi}}{(1+r_e)^2} + \frac{1}{2} \cdot \frac{1-r_e}{1+r_e} \right|^2 + \frac{1}{2} \left[\frac{1-r_e}{1+r_e} \right]^2 \quad (29)$$

The first term in the curly brackets of (28) has two contributions: an internally reflected wave and a dielectrically reflected wave generated respectively by the right-handed and left-handed circular components of the incident beam. Since these two reflected waves are right-handed circularly polarized, they are subjected to interference. For $r_e > 1$ constructive interference occurs at the long wavelength edge of the band ($\lambda' = (1+\alpha)^{1/2}$), where $\phi = 180^\circ$ while destructive interference occurs at the other edge ($\lambda' = (1-\alpha)^{1/2}$, $\phi = 0^\circ$). The converse is true for $r_e < 1$. Outside the reflection band, the internally reflected component decreases monotonically and R_p approaches R_{p0} as given by Eq. (10).

The wavelength dependence of R_p as predicted by (20) fits quite well the graphs of Figure 5a. (Note that Eq. (29) is limited to the reflection band.)

While linear polarization in the x direction is a sum of circular polarizations (see Eq. 27), linear polarization in the y direction is their difference. Therefore the reflection coefficient is

$$R_p = 2 \left| \frac{2r_e e^{i\phi}}{(1+r_e)^2} - \frac{1}{2} \cdot \frac{1-r_e}{1+r_e} \right|^2 + \frac{1}{2} \cdot \left[\frac{1-r_e}{1+r_e} \right]^2 \quad (30)$$

and for $r_e > 1$, the two terms inside the absolute value sign, will interfere constructively at the short wavelength edge of the band and destructively at the other band edge (cf. Figure 5b).

We will treat now the case where the incident wave is unpolarized. An unpolarized wave of unit intensity has the following decomposition into circular components:

$$\begin{bmatrix} Ex \\ Ey \end{bmatrix} = \left\{ \frac{1}{2} \cdot \begin{bmatrix} 1 \\ i \end{bmatrix} + \frac{1}{2} e^{i\theta} \cdot \begin{bmatrix} 1 \\ -i \end{bmatrix} \right\} \exp \left[2\pi i \left(\frac{t}{T} - \frac{nz}{\lambda} \right) \right] \quad (31)$$

θ is a randomly distributed relative phase. The reflected wave in the band is given by:

$$\begin{bmatrix} Ex \\ Ey \end{bmatrix} = \left\{ \left[\frac{2r_\epsilon e^{i\phi}}{(1+r_\epsilon)^2} + \frac{1}{2} e^{i\theta} \cdot \frac{1-r_\epsilon}{1+r_\epsilon} \right] \cdot \begin{bmatrix} 1 \\ -i \end{bmatrix} + \frac{1}{2} \cdot \frac{1-r_\epsilon}{1+r_\epsilon} \cdot \begin{bmatrix} 1 \\ i \end{bmatrix} \right\} \exp \left[2\pi i \left(\frac{t}{T} + \frac{nz}{\lambda} \right) \right] \quad (32)$$

Because θ is randomly distributed

$$Rp = 2 \left| \frac{2r_\epsilon e^{i\phi}}{(1+r_\epsilon)^2} \right|^2 + \frac{1}{2} \left[\frac{1-r_\epsilon}{1+r_\epsilon} \right]^2 + \frac{1}{2} \left[\frac{1-r_\epsilon}{1+r_\epsilon} \right]^2 = \frac{1+6r_\epsilon^2+r_\epsilon^4}{(1+r_\epsilon)^4} \quad (33)$$

Thus Rp is constant throughout the band and approaches Rp_0 for wavelengths far from the band.

We shall turn our attention now to some experimental considerations. The values for α and r_ϵ which we have used are attainable¹¹ so that Figures 4 and 5 can be experimentally tested. In any reflection experiment the liquid crystal is contained in a cell usually between two glass plates. Thus both the glass and the liquid crystal are finite while the above figures have been obtained for the semi-infinite glass, semi-infinite liquid crystal case. But the use of wedge-shaped plates (this is only one way to avoid the air-glass dielectric reflection) and "thick" liquid crystal slabs,^{3,16} ensures us that the resulting spectra will have the same shape as in Figures 4 and 5 with one important difference: all the sharp corners will be smoothed. We have calculated theoretically the behavior of a finite slab assuming that the two interfaces are parallel forming a Fabry-Perot cavity. This assumption results in interference due to multiple reflections between the interfaces.⁵ Examples of such calculated spectra appear in Figures 7a and 7b. Interference appears in these graphs in the form of a fine structure. It should be noted that experimentally the condition of parallelism is not usually fulfilled so that this fine structure is smeared (for an exception, see Ref. 10).

V CONCLUSION

In this paper we have discussed glass-liquid crystal interfaces where the glass' refractive index is not equal to the average index of refraction of the liquid crystal. The reflected spectra for various incident polarizations were calculated and the results show that the shape of the reflection spectrum inside the cholesteric band can be tuned by changing the polarization of the incident wave. Our solutions (17) and (22) can be utilized to compute the reflection coefficient for any arbitrary normally incident wave.

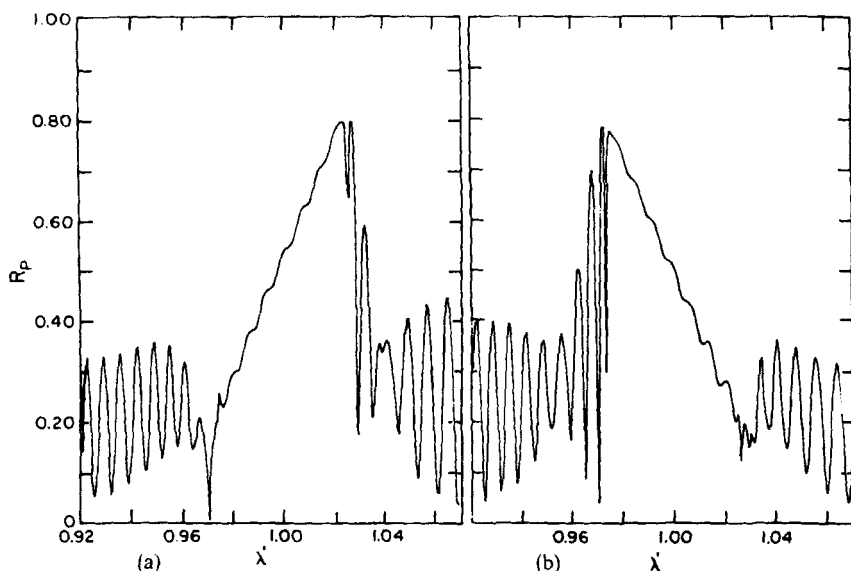


FIGURE 7 Reflection spectra from a finite slab whose thickness equals $64p$.

$\alpha = 0.05$, $r_e = 2$. The incident wave is linearly polarized:

(a) in the x -direction.

(b) in the y -direction.

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14. Intensity is measured here by $|e'_x|^2 + |e'_y|^2$.
15. Note that Eq. (22) is only an approximation. In fact R_p varies slightly within the band (cf. Figure 4a).
16. In our case $\alpha = 0.05$ so that according to Reference 3 "thick" means $d = 40p$ (d is the thickness).
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